

# Investigating Viscosity in Newtonian Fluids

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# Abstract

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The aim of this investigation is to evaluate the viscosities of three different liquids, using three different methods and examining the results to determine the most effective and accurate method. The different methods used were Stokes' Law, at first varying the liquid used, and in experiment 2 the effect of varying the diameter of the ball was examined. The third method used to measure the viscosity of the liquids was by using a viscometer, which proved to be the least effective method. The three liquids examined were Glycerol, Honey and Castor oil.

## Findings

It was found that the liquid with the highest viscosity was honey, second was Glycerol and the least viscous was Castor oil. Furthermore, the most effective and accurate method at determining these viscosities was Stokes' law, as this method produced the values closest to their standard values. Additionally, the validity of this experiment could be examined further by investigating the effect of varying the ball size, which validated the result obtained for the viscosity of glycerol in experiment 1.

In the first experiment, the viscosity values obtained were:

Glycerol  $\eta = 30.57 \pm 0.74$  Poise at  $25 \pm 0.1$  °C

Honey  $\eta = 113.02 \pm 9.57$  Poise at  $28.2 \pm 0.1$  °C

Castor oil  $\eta = 21.32 \pm 0.37$  Poise at  $25 \pm 0.1$  °C

Furthermore, the second experiment gave similar, but less accurate values for the viscosities of each liquid.

Glycerol  $\eta = 26.04 \pm 0.98$  Poise at  $\pm 0.1$  °C

Finally, experiment 3 proved to be the least effective method for measuring viscosity. This could be due to the poor calibration of the viscometer, as it took a few times for it the upper reservoir to drain properly, and the viscometer was not washed each time, hence liquid residue could have affected the results. The results obtained in this experiment were:

Glycerol  $\eta = 10.97$  Poise at 28.3 °C

Honey  $\eta = 27.73$  Poise at 28.3 °C

Castor Oil  $\eta = 8.4$  Poise at 28.3 °C

$21.32 \pm 0.37$  Poise at  $25 \pm 0.1$  °C

# Underlying Physics

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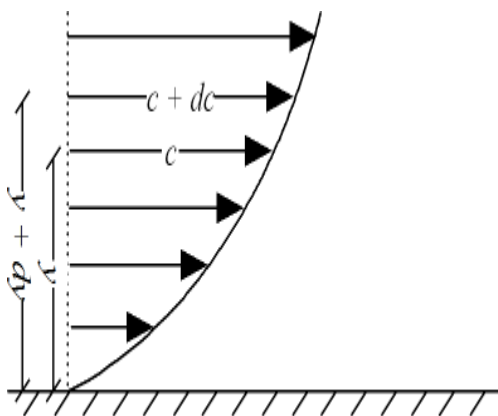
Viscosity is the resistance of a fluid to flow or its resistance to shear stress. Furthermore, viscosity is determined by the intermolecular forces within a substance, that is, the forces acting between molecules. This is because, for a fluid to flow, the intermolecular forces must break, hence, the more (or stronger) intermolecular forces, the greater the fluid's viscosity.

## *Dynamic Viscosity*

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Dynamic Viscosity is the ratio of the shearing stress  $\frac{\vec{F}}{A}$  (or  $\tau$ ) to the velocity gradient in a fluid,

$$\frac{\vec{F}}{A} = \eta \frac{dv_x}{dz}$$



www.engineeringtoolbox.com

This expression, known as Newton's Law of Viscosity, states that the resulting shear of a fluid is directly proportional to the force applied and has an inverse relationship with its viscosity.

In the formula,

$\vec{F}$  Is the force

$A$  Is the Area the Force acts upon

$\eta$  Is the fluid's dynamic viscosity.

$\frac{dv_x}{dz}$  Is the velocity gradient.

Dynamic (absolute) viscosity takes place when one molecular layer of a fluid slides over another layer, causing internal friction. Absolute viscosity is measured in Poise (more commonly Centi-Poise) after the French Physiologist Jean Louis Poiseuille. One Pascal Second is equal to ten Poise.

([engineeringtoolbox.com](http://engineeringtoolbox.com))

## *Kinematic Viscosity*

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Kinematic viscosity ( $\nu$ ) is the ratio of a fluid's dynamic viscosity to its density, measured in  $cm^2 s^{-1}$ , more commonly called a Stoke.

$$\nu = \frac{\eta}{\rho}$$

Kinematic viscosity is also the effect that gravity has on a viscous liquid. Moreover, it can be measured by pouring two liquids of different viscosities from a given height into a container below and measuring the time for them to reach the bottom of that container. The fluid with the greatest kinematic viscosity will take the longest time to reach the bottom.

(Batchelor G.K, 1967)

## *Stokes' Law*

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George Gabriel Stokes was an Irish Mathematician who graduated from Cambridge University as the Senior Wrangler (the top student graduating with a first class degree). Stokes then went on to obtain the role as Lucasian Professor at Cambridge where he conducted research on light polarization, Engineering and fluid dynamics. In 1851, Stokes derived an equation for the frictional (drag) force exerted on a sphere with a small Reynolds number in a continuous viscous flow, now called, Stokes' Law. Simply stated, Reynold's number gives a measure of the ratio of inertial forces to viscous forces. To elaborate, Reynold's number is a measure of the rate of movement of a fluid to the resistance of flow (viscosity). Therefore, a fluid with a small Reynold's number suggests that the fluid has high viscosity.

[this relationship is valid only for 'laminar' flow. Laminar flow is defined as a condition where fluid particles move along in smooth paths in lamina (fluid layers .sliding over one another).

$$Re = \frac{\rho \vec{v} l}{\eta}$$

Where  $\rho$  is density,  $\vec{v}$  is mean velocity,  $L$  is the travelled length of the fluid and  $\eta$  is the dynamic viscosity.

In the experiment conducted by Stoke, a sphere was placed in a viscous liquid and allowed to reach terminal velocity. The sphere reached terminal velocity as the drag force created by its motion is proportional to the square of its speed. Therefore, an object travelling slowly will create a Stokes drag force which is less than the gravitation force so the object accelerates. At it accelerates, the Stokes drag increases, until it is equal to the weight of the ball.

The time for the ball to pass through a known distance of fluid was then measured. In doing this, Stoke found that the frictional force due to the sphere flowing through a continuous fluid is expressed as follows

$$F_d = 6\pi\eta r \vec{v}_s$$

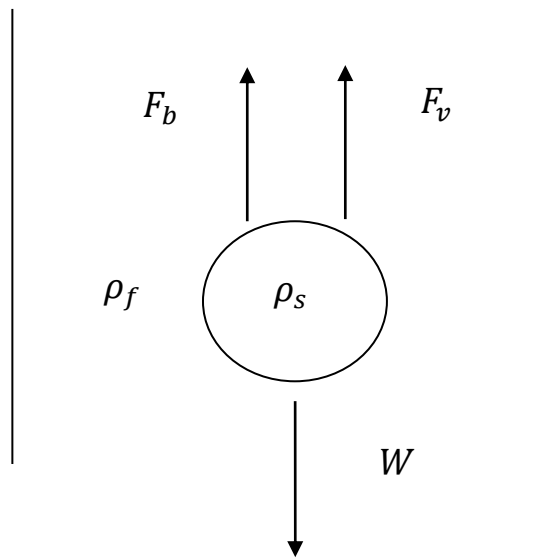
$F_d$  Is the frictional force between the liquid and the sphere known as Stokes' drag.

$\eta$  Is the fluid's dynamic viscosity.

$r$  Is the radius of the sphere.

$\vec{v}_s$  Is the terminal velocity of the sphere in the fluid.





The diagram shows the Buoyant force  $F_v$ , Stokes' drag  $F_b$  and the force due to gravity  $W$ . Additionally,  $\rho_s$  and  $\rho_f$  are the densities of the sphere and the fluid, respectively.

By vector addition,  $W = F_b + F_v$  (1)

Additionally, since the mass of the sphere is equal to the product of its volume and density,

$$m = \frac{4}{3} \pi r^3 \rho_s$$

the weight of the sphere can be written as,

$W = mg = \frac{4}{3} \pi r^3 \rho_s g$  Where  $m$  is the mass,  $r$  is the radius of the ball,  $\rho_s$  is the density of the sphere and  $g$  is the acceleration due to gravity ( $9.81 \text{ ms}^{-2}$ ).

Similarly, as the ball is allowed to reach terminal velocity, by Newton's first law, the net force acting on it will equal zero, hence the force  $F_b$  will be equal and opposite to  $W$ . Additionally, by Archimedes' principle that, the upward buoyant force exerted on a body immersed in a fluid is equal to the weight of the fluid the body displaces, this suggests that the ball displaces exactly the same volume of fluid as the volume it contains, therefore,

$$F_b = \frac{4}{3}\pi r^3 \rho_l g$$

In this case, the density of the liquid ( $\rho_l$ ) is taken into account instead.

Furthermore, substituting these equations into equation (1), and rearranging for the viscous force ( $F_b$ ), gives

$$F_b = \frac{4}{3}\pi r^3 g (\rho_s - \rho_l)$$

Then, by using Stokes' drag equation,

$$6\pi\eta R \vec{v}_s = \frac{4}{3}\pi r^3 g (\rho_s - \rho_l)$$

Hence, rearranging for dynamic viscosity and letting  $\vec{v}_s = \frac{l}{t}$  gives,

$$\eta = \frac{2(\rho_s - \rho_l)}{9} \frac{r^2 g t}{l}$$

Where  $l$  is the distance travelled by the ball in the fluid and  $t$  is the time taken to do so.

Therefore, using this equation, the viscosity of a fluid can be calculated.

# Stokes' Law

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## *Experiment 1 – Varying The Fluid Used*

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### *Aim*

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The aim of this experiment is to measure the viscosity of three different liquids using Stokes' Law.

### *Apparatus*

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Ball-bearing

Stop watch

Thermometer

3 Liquids

Micrometer

Vernier Calipers

Magnet

Pen

Funnel



## *Procedure*

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A steel ball was dropped into a 100  $cm^2$  cylinder from the centre of its rim, filled with a viscous liquid, and the time measured for the ball to pass two points marked on the tube at 90  $cm^2$  and 20  $cm^2$  with a black board pen. Three different tubes were used with the different liquids to ensure no cross-contamination of liquids. The three liquids used were Glycerol, Honey and Castor oil and the same ball was used with each liquid.

Additionally, Extra liquid was allowed above the 100  $cm^2$  to ensure that the ball would reach its terminal (maximum) velocity, before reaching the 90  $cm^2$  mark. When the ball passed the 90  $cm^2$  mark, the timer started, and when it reached the 20  $cm^2$ , the timer was stopped and the result recorded. This was repeated five times to ensure an accurate reading and so that an average could be calculated. So that the viscosity of the liquid could be calculated, the diameter of the sphere was measured, at first using the calipers, gripping so the ball could still pass through its grip, then confirmed using a micrometer for a more accurate measurements. Since viscosity is temperature dependent, a thermometer measured the room temperature before and after the experiment, and was hoped to remain constant. A large magnet was used to retrieve the ball from the bottom of the tube.

Finally, to calculate the viscosity of the liquid, the density of it must be known. To find this, the mass of an empty measuring cylinder was weighed three times using a balance. The liquid was then poured into the cylinder at three different known volumes and weighed for each volume. Additionally, the mass of each balled used was measured using the balance. From this, the mass of the liquid was measured and hence the density could be calculated.

## *Results*

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### ***Propan-1, 2, 3 – triol (Glycerol)***

Room Temperature before experiment: 25.8 °C

Diameter of ball bearing using the micrometer: 3.533mm, 3.534mm, 5.534mm

Diameter of ball bearing using the calipers: 4mm, 4mm, 4mm

Volume of liquid that the ball fell through: upper mark – lower mark

90mm-20mm = 70mm

Time to fall through distance:

1. 3.78 s
2. 3.68 s
3. 3.82 s
4. 3.72 s
5. 3.84 s
6. 3.67 s
7. 3.83 s
8. 3.93 s
9. 3.87 s
10. 3.66 s

Final Room Temperature: 25.1 °C

Average time for ball to fall marked distance:

$$\bar{t} = \frac{3.78 + 3.68 + 3.82 + 3.72 + 3.84 + 3.67 + 3.83 + 3.93 + 3.87 + 3.66}{10}$$

$$\therefore \bar{t} = 3.78 \text{ s}$$

Average Diameter of Ball Bearing Using Micrometer:

$$\frac{3.83 + 3.84 + 3.84}{3} = 3.84mm$$

Average Diameter of Ball Bearing Using Callipers:

$$\frac{4 + 4 + 4}{3} = 4mm$$

Hence the average diameter of the sphere is,

$$\frac{(3.84 + 4) \times 10^{-3}}{2} = 3.92 \text{ mm}$$

∴ The Average radius = 1.96 mm

Standard density of steel is  $7.8 \times 10^3 \text{ kg m}^{-3}$

To calculate the density of the liquid, the mass and volume must be known. To improve the accuracy of this calculation, multiple volumes were used to obtain different masses and an average of each taken.

Volume of liquid in cylinder:  $100 \text{ ml} = 1 \times 10^{-4} \text{ m}^3$

Mass of empty cylinder: 39.99g, 40.03g, 40.03g

$$\text{Average mass of cylinder} = \frac{39.99 + 40.03 + 40.03}{3} = 40.02 \text{ g}$$

Mass of filled cylinder: 166.94 g, 166.96 g, 166.95 g

$$\text{Average mass of filled cylinder } (\bar{m}): = \frac{166.94 + 166.96 + 166.95}{3} = 166.95$$

$$\therefore \text{mass of liquid } (m_l) = 166.95 - 40.02 = 126.93 \text{ g}$$

$$= 0.13 \text{ kg}$$

$$\text{Density of liquid } (\rho_l) = \frac{0.13}{1 \times 10^{-4}}$$

$$= 1300 \text{ kg m}^{-3}$$

Volume:  $50 \text{ ml} = 0.5 \times 10^{-4} \text{ m}^3$

Mass of cylinder and liquid: 103.15 g, 103.16 g, 103.16 g

$$\bar{m} = \frac{103.15 + 103.16 + 103.16}{3} = 103.16 \text{ g}$$

$$\therefore m_l = (103.16 - 40.02) \times 10^{-3} = 0.06 \text{ kg}$$

$$\rho_l = \frac{0.06}{0.5 \times 10^{-4}} = 1200 \text{ kg m}^{-3}$$



Volume:  $25\text{ml} = 0.25 \times 10^{-4}\text{m}^3$

Mass of cylinder and liquid: 74.75 g, 74.76 g, 74.75 g

$$\bar{m} = \frac{74.75 + 74.76 + 74.75}{3} = 74.75 \text{ g}$$

$$\therefore m_l = (74.75 - 40.02) \times 10^{-3} = 0.036 \text{ kg}$$

$$\rho_l = \frac{0.036}{0.25 \times 10^{-4}}$$

$$= 1429.2 \text{ kgm}^{-3}$$

$$\text{Average Density } (\bar{\rho}_l) = \frac{1300+1200+1429.2}{3} = 1309.73 \text{ kg m}^{-3}$$

Using the expression derived from Stokes' law, the viscosity of Glycerol can be calculated.

$$\eta = \frac{2(\rho_s - \rho_l)}{9} \frac{r^2 g t}{l}$$

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1309.73)}{70 \times 10^{-3}} (1.96 \times 10^{-3})^2 (9.81)(3.78)$$

$$\therefore \eta = 3.057 \text{ Pa s}$$

$$\eta = 30.57 \text{ Poise at } 25 \pm 0.1 \text{ }^\circ\text{C}$$

## *Honey*

Room Temperature before experiment: 28.2 °C

Time to fall through distance:

1. 15.28 s
2. 15.22 s
3. 15.85 s
4. 17.5 s
5. 14.6 s
6. 13.53 s
7. 18.34 s
8. 14.38 s
9. 13.5 s
10. 19.16s

Final Room Temperature: 28.3 °C

$$\bar{t} = 15.73 \text{ s}$$

Volume of liquid in cylinder:  $100 \text{ ml} = 1 \times 10^{-4} \text{ m}^3$

Mass of cylinder: 40.53 g, 40.53 g, 40.53 g

$$\bar{m}_c = \frac{40.53 + 40.53 + 40.53}{3} = 40.53 \text{ g}$$

Mass of filled cylinder (g): 179.67, 179.62, 179.68

$$\bar{m} = 179.66$$

$\therefore$  mass of liquid ( $m_l$ ) =  $179.66 - 40.53 = 139.13 \text{ g}$

= 0.19 kg

$$\rho_l = \frac{0.19}{1 \times 10^{-4}}$$
$$= 1931.3 \text{ kgm}^{-3}$$

Volume =  $0.5 \times 10^{-4} \text{ m}^3$

Mass of cylinder and liquid: 116.22 g, 116.21 g, 116.22 g

$$\bar{m} = 116.22 \text{ g}$$

$\therefore m_l = 116.22 - 40.53 = 0.08 \text{ kg}$

$$\rho_l = \frac{0.08}{0.5 \times 10^{-4}}$$

=  $1513.8 \text{ kgm}^{-3}$

Volume:  $25\text{ml} = 0.25 \times 10^{-4}\text{m}^3$

Mass of cylinder and liquid: 88.97 g, 88.96 g, 88.97 g

$$\bar{m} = 88.97 \text{ g}$$

$$\therefore m_l = (88.97 - 40.53) \times 10^{-3} = 0.0484 \text{ kg}$$

$$\rho_l = \frac{48.44 \times 10^{-3}}{0.25 \times 10^{-4}}$$

$$\rho_l = 1937.6 \text{ kgm}^{-3}$$

$$\bar{\rho}_l = \frac{1931.3 + 1513.8 + 1937.6}{3} = 1794.23 \text{ kg m}^{-3}$$

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1794.23)}{70 \times 10^{-3}} (1.96 \times 10^{-3})^2 (9.81)(15.73)$$

$$\therefore \eta = 11.3 \text{ Pa s}$$

$$\eta = 113.02 \text{ Poise at } 28.2 \pm 0.1 \text{ }^\circ\text{C}$$

## *Castor Oil*

Initial Room Temperature: 28.3°C

Timings:

1. 2.12 s
2. 2.63 s
3. 3.69 s
4. 2.19 s
5. 3.81 s
6. 2.5 s
7. 2.06 s
8. 2.18 s
9. 3.08 s
10. 2.15 s

Final Room Temperature: 28.3 °C

$$\bar{t} = 2.64 \text{ s}$$

Volume of liquid in cylinder:  $100 \text{ ml} = 1 \times 10^{-4} \text{ m}^3$

Mass of cylinder (g): 41.2 , 41.19 , 41.2

$$\bar{m}_c = 41.2 \text{ g}$$

Mass of filled cylinder (g): 137.52, 137.51, 137.51

$$\bar{m} = 137.51 \text{ g}$$

$$\therefore \text{mass of liquid } (m_l) = (137.51 - 41.2) \times 10^{-3} = 0.096 \text{ kg}$$

$$\rho_l = \frac{0.096}{1 \times 10^{-4}}$$

$$\rho_l = 963.1 \text{ kgm}^{-3}$$

Volume =  $0.5 \times 10^{-4} \text{ m}^3$

Mass of cylinder and liquid (g): 92.1, 92.1, 92.09

$$\bar{m} = 92.1 \text{ g}$$

$$\therefore m_l = (92.1 - 41.2) \times 10^{-3} = 0.05 \text{ kg}$$

$$\rho_l = \frac{0.05}{0.5 \times 10^{-4}}$$

$$\rho_l = 1018 \text{ kgm}^{-3}$$

Volume:  $25\text{ml} = 0.25 \times 10^{-4}\text{m}^3$

Mass of cylinder and liquid (g): 70.14, 70.13, 70.13

$$\bar{m} = 70.13 \text{ g}$$

$$\therefore m_l = (70.13 - 41.2) \times 10^{-3} = 0.029 \text{ kg}$$

$$\rho_l = \frac{0.029}{0.25 \times 10^{-4}}$$

$$= 1160 \text{ kgm}^{-3}$$

$$\bar{\rho}_l = \frac{963.1 + 1018 + 1160}{3} = 1047.03 \text{ kg m}^{-3}$$

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1047.03)}{70 \times 10^{-3}} (1.96 \times 10^{-3})^2 (9.81)(2.64)$$

$$\therefore \eta = 2.132 \text{ Pa s}$$

$$\eta = 21.32 \pm 0.37 \text{ Poise at } 28.2 \pm 0.1 \text{ }^\circ\text{C}$$

## *Uncertainties*

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Thermometer ( $^{\circ}\text{C}$ ):

Scale Reading =  $\pm 0.1$

Calibration =  $\pm 0.1$

$$\therefore \Delta T = \pm 0.1 \text{ } ^{\circ}\text{C}$$

Where  $\Delta T$  is the absolute uncertainty of temperature. The symbol  $\Delta$  will be used to denote the absolute uncertainty.

Micrometer ( $D_m$ ) (mm):

Scale Reading =  $\pm 0.005$

Calibration =  $\pm 0.002$

$$\Delta D_m^2 = (0.005)^2 + (0.002)^2$$

$$\Delta D_m = \pm 0.005 \text{ mm}$$

Callipers:

Scale Reading =  $\pm 0.025 \text{ mm}$

Calibration =  $0.01 \text{ mm}$

$$\Delta D^2 = 0.025^2 + 0.01^2$$

$\Delta D$  0.0274 mm

$$\therefore \Delta r = 0.014 \text{ mm}$$



Measuring Cylinder:

Scale reading =  $\pm 0.5$  mm

$$\Delta l = \pm 0.5 \text{ mm}$$

$$\therefore \Delta V = \pm 0.5 \text{ ml}$$

Timer:

$$\Delta t = \pm 0.01 \text{ s}$$

Balance:

Scale reading =  $\pm 0.005$  g

Calibration =  $\pm 0.01$  g

$$\Delta m = 0.011 \text{ (By Pythagoras)}$$

Uncertainty of density of sphere:

$$\rho = \frac{m}{V}$$

Hence, since  $\frac{4}{3}$  and  $\pi$

$$\rho \propto \frac{m}{r^3}$$

Radius of sphere using micrometer =  $3.83 \pm 0.014$  mm

% Uncertainty = 0.26 %

$$\bar{m} = 0.24 \pm 0.01 \text{ g}$$

% Uncertainty = 4.16 %

Using largest percentage uncertainty (4.16%),

$$\Delta \rho_s = 4.16 + 3(0.26) = 4.94 \%$$

$$\therefore \Delta \rho_s = \pm 385.32 \text{ kgm}^{-3}$$

## ***Glycerol***

Volume = 100ml

$$\left(\frac{\Delta\rho_1}{\rho_1}\right)^2 = \left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2$$

$$\left(\frac{\Delta\rho_1}{1300}\right)^2 = \left(\frac{0.011}{0.13}\right)^2 + \left(\frac{0.5 \times 10^{-4}}{1 \times 10^{-4}}\right)^2$$

$$\therefore \Delta\rho_1 = \pm 659.24 \text{ kgm}^{-2}$$

Volume = 50 ml

$$\Delta\rho_2 = \pm\rho\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}$$

$$\Delta\rho_2 = \pm 1220 \text{ kgm}^{-3}$$

Volume = 25 ml

$$\Delta\rho_3 = \pm 2891.57 \text{ kgm}^{-3}$$

$\therefore$  The absolute uncertainty in density of Glycerol is,

$$\Delta\rho^2 = (659.24)^2 + (1120)^2 + (2891.57)^2$$

$$\therefore \Delta\rho = \pm 3170.2 \text{ kgm}^{-3}$$

Absolute uncertainty in viscosity

$$\left(\frac{\Delta\eta}{\eta}\right)^2 = \left(\frac{\Delta\rho_s + \Delta\rho_l}{\rho_s + \rho_l}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(2 \times \frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta t}{\bar{t}}\right)^2$$

$$\left(\frac{\Delta\eta}{30.57}\right)^2 = \left(\frac{385.32 + 317.02}{7.8 \times 10^3 + 1309.73}\right)^2 + \left(\frac{0.5}{70}\right)^2 + \left(2 \times \frac{0.014}{1.96}\right)^2 + \left(\frac{0.01}{3.78}\right)^2$$

$$\Delta\eta = 2.41\%$$

$$\eta = 30.57 \pm 0.74 \text{ Poise at } 25 \pm 0.1 \text{ }^\circ\text{C}$$

## ***Honey***

Volume = 100 ml

$$\Delta\rho_1 = 1931.3 \sqrt{\left(\frac{0.011}{0.19}\right)^2 + \left(\frac{0.5 \times 10^{-4}}{1 \times 10^{-4}}\right)^2}$$

$$\Delta\rho_1 = \pm 972.1 \text{ kgm}^{-3}$$

$$\Delta\rho_2 = \pm 1528.04 \text{ kgm}^{-3}$$

$$\Delta\rho_3 = \pm 3900.14 \text{ kgm}^{-3}$$

$$\Delta\rho = \sqrt{(972.1)^2 + (1528.04)^2 + (3900.14)^2}$$

$$\Delta\rho = 4300.11 \pm \text{kgm}^{-3}$$

$$\left(\frac{\Delta\eta}{113.02}\right)^2 = \left(\frac{385.32 + 317.02}{7.8 \times 10^3 + 1794.23}\right)^2 + \left(\frac{0.5}{70}\right)^2 + \left(2 \times \frac{0.014}{1.96}\right)^2 + \left(\frac{0.01}{15.73}\right)^2$$

$$\Delta\eta = 8.47 \%$$

$$\eta = 113.02 \pm 9.57 \text{ Poise at } 28.2 \pm 0.1 \text{ }^\circ\text{C}$$

### *Castor Oil*

$$\Delta\rho_1 = \pm 494.03 \text{ kgm}^{-3}$$

$$\Delta\rho_2 = \pm 1042.34 \text{ kgm}^{-3}$$

$$\Delta\rho_3 = \pm 2361.36 \text{ kgm}^{-3}$$

$$\Delta\rho = \pm 2628.03 \text{ kgm}^{-3}$$

$$\Delta\eta$$

$$= (21.32) \sqrt{\left(\frac{385.32 + 317.02}{7.8 \times 10^3 + 1047.03}\right)^2 + \left(\frac{0.5}{0.7}\right)^2 + \left(2 \times \frac{0.014}{1.96}\right)^2 + \left(\frac{0.01}{2.64}\right)^2}$$

$$\Delta\eta = 1.73 \%$$

$$\eta = 21.32 \pm 0.37 \text{ Poise at } 28.3 \pm 0.1 \text{ }^\circ\text{C}$$

(Squires G.L, 2001)

## *Evaluation*

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The results obtained in this experiment show that honey (113.02 Poise) is the most viscous, followed by Glycerol (30.57 Poise) and that Castor oil (21.32 Poise) is the least viscous.

However, the results from this experiment are not exactly equal to the standard values. For example, the standard viscosity of Glycerol is 10 Poise, which is far less than the value obtained by my calculations. Because of this, it is clear that uncertainties and practical errors played a large factor in the results. (engineering toolbox.com)

Hence, the results of this experiment could be improved had such errors been minimised. These errors include: the ball sliding down the side of the cylinder as it descends, this slowed the ball as it created more unnecessary friction, to fix this problem, a filter funnel or dropping device should have been used to ensure the ball be dropped in the centre of the measuring cylinder.

Similarly, the ball should have been thoroughly cleaned before each drop, minimising residue and hence minimising unnecessary friction. Additionally, the temperature of the room varied during the experiment due to windows being left opened, to fix this, the experiment should be done in a room where the temperature is constant.

To continue, parallax uncertainty played part in the micrometer and caliper uncertainties as I was judging the lining up of lines to measure the ball's diameter. Similarly, the sphere's radius was not constant, this was proven by taking multiple measurements and using two pieces of measuring equipment. Although taking multiple readings decreases the uncertainty in its measurement, significantly more readings should be taken to ensure that an uneven surface area plays a minor role in the overall uncertainty of the experiment.

## *Experiment 2 – Varying The Ball Size*

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### Aim

The aim of experiment 2 is to examine the effect that varying the ball size used has on measuring the viscosity of a liquid using Stokes' law sized balls

### Apparatus

3 different sized ball-bearings

Stop watch

Thermometer

1 Liquid

Micrometer

Magnet

pen

### Procedure

The set up was the same as the previous experiment, but with only one fluid (Glycerol) and the same volume as previous, being used (approximately  $100\text{ cm}^3$ ). However, in this exercise, three different sized balls were used. Each ball was measured three times at three different points on the ball's surface, using the micrometer. As before, the initial and final room temperatures were measured using a thermometer. As before, each ball was dropped into the liquid 10 times and the time taken for the ball to fall between the  $90\text{ cm}^3$  and the  $20\text{ cm}^3$  marks was timed.

## Results

### ***Ball 1***

Diameter using micrometer (mm): 5.39, 5.27 5.3

Hence, taking the average diameter and halving gives,

$$\bar{r} = 2.66\text{mm}$$

Times for ball to fall 70  $\text{cm}^3$  (s):

1. 2.09
2. 2.09
3. 2.16
4. 1.94
5. 2.00
6. 1.97
7. 1.90
8. 1.91
9. 1.90
10. 1.87

$$\bar{t} = 1.98 \pm 0.01 \text{ s}$$

Room Temperature before experiment:  $25.8 \pm 0.1$  °C

Final Room Temperature:  $25.8 \pm 0.1$  °C

Using information from *experiment* (see pages 13-14)1, the viscosity of glycerol can be calculated as such.

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1309.73)}{70 \times 10^{-3}} (2.66 \times 10^{-3})^2 (9.81)(1.98)$$

$$\therefore \eta = 28.32 \text{ Poise at } 25.8 \pm 0.1^\circ\text{C}$$



## ***Ball 2***

Diameter using micrometer (mm): 3.83, 3.83, 3.84

$$\bar{r} = 1.92 \text{ mm}$$

Timings (s):

1. 3
2. 3
3. 2.97
4. 2.81
5. 2.81
6. 2.69
7. 2.72
8. 2.65
9. 2.69
10. 2.78

$$\bar{t} = 2.81 \pm 0.01 \text{ s}$$

Room Temperature before experiment:  $25.9 \pm 0.1 \text{ }^\circ\text{C}$

Final Room Temperature:  $28.8 \pm 0.1 \text{ }^\circ\text{C}$

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1309.73)}{70 \times 10^{-3}} (1.92 \times 10^{-3})^2 (9.81)(2.81)$$

$\therefore \eta = 20.94 \text{ Poise at } 28.8 \pm 0.1 \text{ }^\circ\text{C}$

### ***Ball 3***

Diameter using micrometer (mm): 5.87, 5.88, 5.87

$$\bar{r} = 2.94 \text{ mm}$$

Timings (s):

1. 1.53
2. 1.44
3. 1.50
4. 1.59
5. 1.47
6. 1.50
7. 1.38
8. 1.41
9. 1.34
10. 1.59

Initial room Temperature: 28.8 °C

Final room Temperature: 28.9 °C

$$\bar{t} = 1.48 \pm 0.01 \text{ s}$$

$$\eta = \frac{2}{9} \times \frac{(7.8 \times 10^3 - 1309.73)}{70 \times 10^{-3}} (2.94 \times 10^{-3})^2 (9.81) (1.48)$$

$$\therefore \eta = 25.87 \text{ Poise at } 28.9 \pm 0.1^\circ\text{C}$$

Therefore, the average of these viscosities is,

$$\bar{\eta} = 26.04 \text{ Poise at approximately } 25\text{-}28 \pm ^\circ\text{C}$$

## Uncertainties

---

Random uncertainty in the diameter:

$$\Delta D = \frac{8.84 - 3.83}{3} = \pm 0.003mm$$

$$\therefore \Delta r = \pm 0.00015mm$$

Random uncertainty in the Viscosity:

$$\Delta\eta_{Random} = \frac{28.8 - 25.87}{3} = 0.98 \text{ Poise}$$

Absolute uncertainty in viscosity =  $\pm 0.37$  Poise at  $28.3 \pm 0.1$  °C

As the absolute uncertainty in the viscosity is calculated for a different temperature, the reliability of this result will vary.

Since the largest uncertainty is in the random uncertainty, this is the value used.

Therefore, the viscosity of Glycerol calculated in experiment 2 is,

$$\bar{\eta} = 26.04 \pm 0.98 \text{ Poise at approximately } 25-28 \pm 0.1 \text{ °C}$$

## *Evaluation*

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The results obtained in experiment 2 were similar to the value obtained in experiment 1, 30.57 *Poise*. However, a more accurate measurement could be taken if all of the values were averaged and the uncertainty between them calculated.

Furthermore, the temperature in experiment 2 was greater than in experiment 1 and also fluctuated more, creating an uncertainty that could not be calculated. Additionally, increasing the ball size increases the Stokes drag, however, this is compensated for by the greater volume of sphere and more importantly displaced liquid, hence the value of viscosity calculated is unaffected. To continue, as with experiment 1, parallax uncertainty factored into the results as all timings were judged by eye.

Similarly, as bigger balls were used, the ball would drop almost instantly to the mark, perhaps not reaching its terminal velocity before hitting the top mark, however, this happened in every reading so the results are still legitimate. Furthermore, as with experiment 1, the ball would often slide down the side of the container, slowing it down, this explains the large spread of results for timing. Occasionally, the ball would slow to such an extent sliding down the side that the ball had to be re-dropped as the timing was far too inaccurate. Additionally, to gain better insight to the effect of varying ball size on Stokes law and calculating viscosity, the same experiment should be carried out with multiple liquids, however I hadn't the time to do so.

## Experiment 3 – Measuring Viscosity Using A Viscometer

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### Aim

The aim of the third experiment is to use a viscometer to measure the viscosities of three different liquids.

### Apparatus

Viscometer

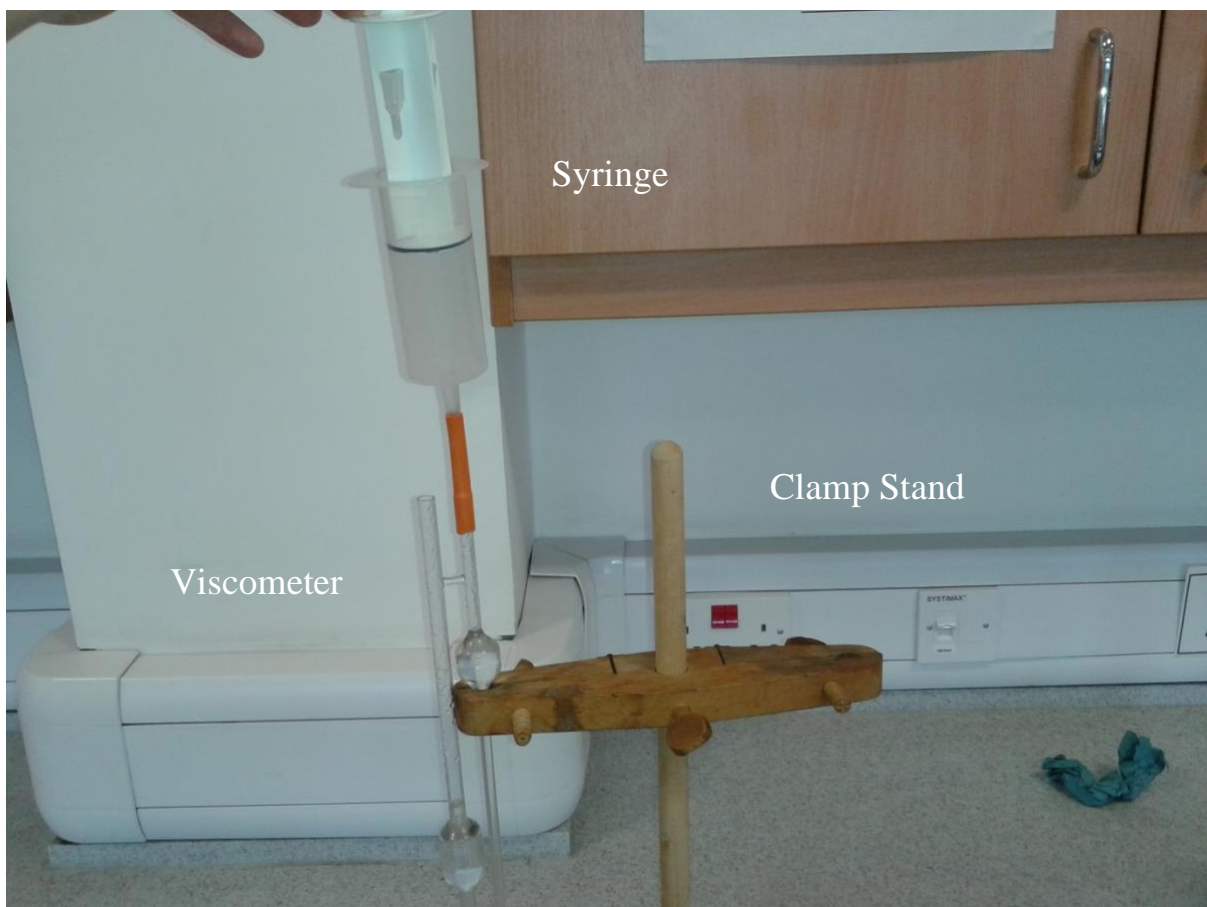
3 Liquids

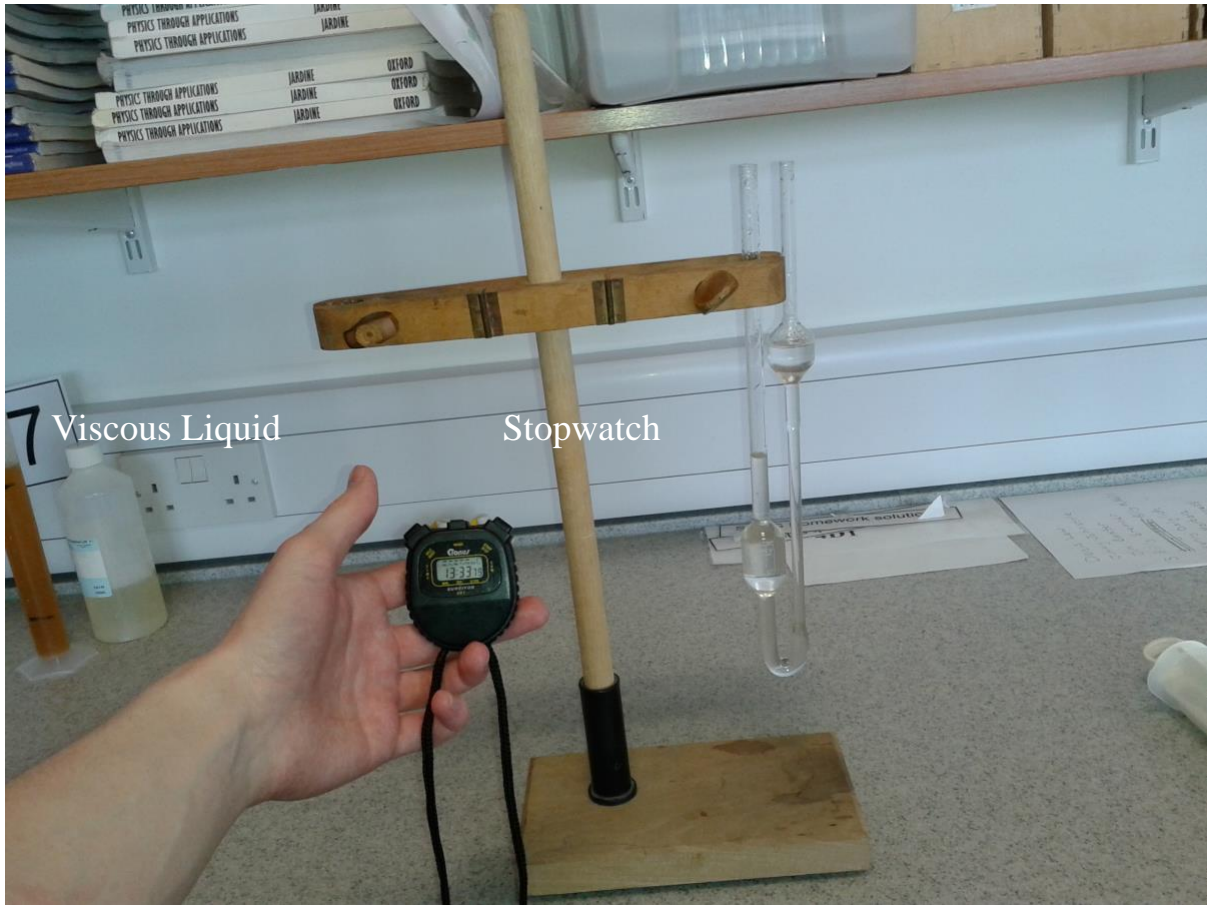
Syringe and piping

Stopwatch

Wooden Clamp stand

Thermometer





7 Viscous Liquid

Stopwatch

## Procedure

A clamp stand is placed on an even surface and the viscometer clamped just below the upper reservoir. The liquid was poured into the thicker end of the viscometer, allowing the liquid to settle with both reservoirs filled almost to the upper marks of each. Normally a syphon is used to pull the liquid up through the viscometer, but I did not have one so one was made using a plastic syringe and piping. This piping was then attached to the syringe and the thin end of the viscometer. Finally, the syringe was pulled and the liquid began to flow through the thinner tube. Eventually, when the liquid reaches just above the upper mark of the upper reservoir, remove the syringe-piping device and when the liquid reaches the upper reservoir mark, the timer was started. The timer was then stopped when the liquid's meniscus flowed to the lower mark on the upper reservoir. This was repeated three times with each liquid.

Additionally, the viscometer did not have a manual so the conversion factor had to be worked out using water. Also, as before the room temperature was measured before and after the experiment to ensure no dramatic temperature change which would affect the viscosity of the liquids. Finally, from the results obtained, the viscosity of the liquid could be calculated using the relationship that .

$$\eta = \text{Conversion Factor} \times \text{time}$$

## Results

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### *Conversion Factor*

#### *Water*

Initial Room Temperature: 28.4 °C

Final Room Temperature: 28.3 °C

Time for water to drain through upper reservoir:

1. 5: 21
2. 5:49
3. 6:27
4. 6:55
5. 6:06

$$\bar{t} = 6:32$$

Viscosity of water =  $1.001673 \times 10^{-3} Pa s$

$$Conversion Factor (CF) = \frac{1.001673 \times 10^{-3}}{0.0655} = 0.0153$$



## *Glycerol*

Initial Room Temperature: 28.4 °C

Final Room Temperature: 28.3 °C

Timings (minutes):

1. 66.53

2. 70.24

3. 78.31

$$\bar{t} = 72.93$$

$$\eta = CF \times t$$

$$\eta = 0.0153 \times 71.93$$

$$\therefore \eta = 10.97 \text{ Poise at } 28.3 \pm 0.1^\circ\text{C}$$

## *Honey*

Initial Room Temperature: 28.4 °C

Final Room Temperature: 28.3 °C

Results:

1. 196.52

2. 162.03

3. 184.09

$$\bar{t} = 181.28$$

$$\Delta\eta = CF \times t$$

$$\Delta\eta = 0.0153 \times 181.28 = 2.77$$

$$\therefore \eta = 27.73 \text{ Poise at } 28.3 \pm 0.1 \text{ } ^\circ\text{C}$$

*Castor oil*

Initial Room Temperature:  $28.4 \pm 0.1^\circ\text{C}$

Final Room Temperature:  $28.3 \pm 0.1^\circ\text{C}$

Timings:

1. 54.41

2. 54.16

3. 56.21

$\bar{t} = 54.93$

$$\eta = 0.0153 \times 54.93$$

$\therefore \eta = 8.4 \text{ Poise at } 28.3^\circ\text{C}$

Uncertainties

Glycerol:

Random Uncertainty in the timings:

$$\Delta t = \frac{78.31 - 66.53}{3} = 4.32 \text{ minutes}$$

$$\therefore t = 72.93 \pm 4.32 \text{ minutes}$$

Honey:

Random Uncertainty in the timings:

$$\Delta t = \frac{196.52 - 162.03}{3} = 11.5 \text{ minutes}$$

$$\therefore t = 181.28 \pm 11.5 \text{ minutes}$$

Castor Oil:

Random Uncertainty in the timings:

$$\Delta t = \frac{66.21 - 54.16}{3} = 4.02 \text{ minutes}$$

$$\therefore t = 54.93 \pm 4.02 \text{ minutes}$$

## Evaluation

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This experiment proved to be unsuccessful in gathering reliable results as none of the viscosities calculated were near the values obtained in experiment 1, which are near their standard values. I believe that there is either a large calibration uncertainty with the equipment or it is broken. I believe this to be the case as it took me multiple tries to get it to work, and when it did, it delivered unreasonable results. To fix this problem, ensure that the manual for the equipment is still in the box and that the equipment is properly calibrated.

# Conclusion

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The results obtained from all three experiments show that the most viscous liquid used was honey with values  $113.02 \pm 9.57$  Poise at  $28.2 \pm 0.1$  °C and 27.73 Poise at 28.3 °C. Similarly, the second most viscous liquid was found to be Glycerol, with the values in each experiment being  $30.57 \pm 0.74$  Poise at  $25 \pm 0.1$  °C,  $26.04 \pm 0.98$  Poise at  $\pm 0.1$  °C and *10.97 Poise at 28.3 °C*. Lastly, the values obtained for the least viscous liquid, Castor oil, were *8.4 Poise at 28.3 °C* and  $21.32 \pm 0.37$  Poise at  $25 \pm 0.1$  °C . It should also be taken into account that the temperatures that each of these values are quoted at are unequal and so this should be factored into the comparisons between the three.

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# Evaluation

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This experiment proved to be effective in investigation the viscosity of the liquids as it correctly met my hypothesis and allowed me to calculate the viscosities of the liquids which were all close to the accepted values for them. However, the experiment could have been improved further for example, the Temperature of the room could have remained constant had I ensured this by monitoring the heating and not doing the experiment near the window. Also apparent was a parallax error, since I had to be eye level with the marks as the ball passed them, but it was difficult to ensure I was looking at the marks from exactly the same position as previous times.

Additionally, to ensure a fair test, the ball was dried every time. This allowed the experiment to be fair, since no residue from the previous timing would be left on the ball, as this would cause an increase in friction, affecting the time for the ball to pass the marks. However, to further minimise the friction, the ball should have been thoroughly cleaned and dried. Furthermore, the room temperature was recorded before and after the experiment as a dramatic change in temperature throughout the experiment would cause variations in the fluids viscosity, as viscosity is temperature dependent.

The second experiment was also a successful method for determining the effect of surface area in contact with the liquid and time taken for the ball to pass the marks. The results show clearly that the larger the ball (smaller surface area) the less time it took for the ball to pass the marks. This is explained by the fact that a larger surface area would mean that more liquid could make contact with the ball, hence a greater friction resulted and the ball was more slowed. The problem arose when the diameter of the ball was greater than [sub in value] as the ball dropped too quickly and the timing depended more heavily on reaction speeds, hence increasing the human error and over all uncertainty.

Similarly, the surface of the sphere was not perfectly even, therefore the radius was different at certain points. To minimize this, multiple readings of the diameter were taken using two different pieces of measuring equipment. However, the calipers were found to be less accurate since they could only measure up to 0.1mm and the lines were judged to line up by the human eye, hence creating the potential for human error, increasing the overall uncertainty of the reading. On the other hand, the micrometer measured to the nearest 0.05mm and produced only a scale reading and calibration uncertainty. Furthermore, more readings could have been taken, minimizing the overall uncertainty and improving the accuracy of the average reading.

To continue, some further considerations would be that a filter funnel should have been used to drop the ball into the liquid to ensure that the ball was dropped from the same height, minimizing the chance of the ball not reaching terminal velocity until it has already passed the top mark.

In summary, had such errors and uncertainties been minimised, the results obtained would have been much closer to their standard values and the values of viscosity for each liquid obtained in each experiment would be less spread out.



# References

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1. Batchelor G.K, 1967, *An Introduction to Fluid Dynamics*, Cambridge University Press, page 218
2. Squires G.L, 2001, *Practical Physics*, Cambridge University Press 4<sup>th</sup> edition, page 361
3. [http://www.engineeringtoolbox.com/dynamic-absolute-kinematic-viscosity-d\\_412.html](http://www.engineeringtoolbox.com/dynamic-absolute-kinematic-viscosity-d_412.html)